

mechanics series

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About the Cover: The design is a black-and-white ink drawing created by F. Ianna as part of the course, "Art for Engineers," at Stevens Institute of Technology. The December '71 issue of Engineering Education carries a broader description and more illustrations of the program.

TEACHING ENGINEERING DYNAMICS

Charles W. Allen

Professor of Mechanical Engineering

Chico State College

During the last decade the content of the typical dynamics course has undergone considerable change. Vector notation is now universal, and some dynamics textbooks are introducing the student to matrix methods. New dynamics textbooks are appearing so frequently that it is often difficult for a dynamics instructor to keep abreast of all the new titles. The degree of sophistication of the textbooks is also increasing as each author tries to add something new.

Dynamics is now frequently offered as a sophomore course rather than at the junior level. Considering the increasing complexity of the textbooks, we realize the implication that we are teaching much more material earlier in the engineering program than was previously the case.

An examination of several of the current first-year physics textbooks shows that the mechanics sections have also increased in breadth and depth of coverage over the last few years. One recent book has such extensive coverage of dynamics that it essentially covers all the material which is included in many of the engineering dynamics texts.

These trends have made presentation of anywhere near the complete textbook material in a one semester, three-unit course very difficult. The background of the students, in spite of the use of sophisticated physics texts, appears to be little better than would be expected from an old fashioned introductory physics course devoted primarily to rectilinear motion of particles. Since the professor usually presents advanced topics at the expense of time ordinarily spent on the elementary topics, the students' comprehension and retention are often poor. After the results of the American Society for Engineering Education (ASEE) National Examination (Singer, 1969) appeared, the author found that instructors at other institutions have also found difficulty with the amount of material to be included in a modern course.

ASEE EVALUATION EXAMINATION

In 1968, the Evaluation Student Committee of ASEE invited participation by engineering schools in a national examination. The problems were to be selected from a master list of 26 problems distributed to each participating school. The writer's institution participated in this examination and, for the dynamics test, selected 10 problems for the two-hour examination. When the data from the

examination were reduced, it was apparent that the Chico State College selection was close to the national popularity ranking. The Chico State average of 44.7% in this national examination compares favorably with the national average of 42.8% and the Pacific Southwest Region average of 36.2%.

The report (Singer, 1969) concludes that although advanced topics are now commonplace in dynamics texts, the examination items selected were largely those of an elementary nature. When an item on an advanced topic was selected, the performance was generally poor.

The results of the ASEE Study have confirmed the author's opinion that much of the advanced material in the current dynamics textbook is not covered or is not well understood by the student.

COURSE CONTENT AND PHILOSOPHY

An outline of the dynamics course as currently taught at Chico State is given in the Appendix. The course is part of a common core for all engineering majors and hence must satisfy several objectives as follows:

1. Provide sufficient background in dynamics so that all students, irrespective of engineering major, can handle simple particle and rigid body problems likely to be encountered as part of civil, electrical or mechanical systems.
2. Provide sufficient background for the subsequent course in mechanical vibrations.
3. Prepare the prospective graduate student for entry into a course in advanced dynamics.
4. Give the student sufficient problem solving ability to enable him to pass the EIT examination.

All of the above objectives with the possible exception of the third can be met by a relatively simple course in dynamics. However, the tendency nowadays is to present a general theory and then to apply this to certain simplified problems.

The author was formerly a firm advocate of this approach but recently has modified it to the extent that a general theory is presented for much of the particle kinematics and kinetics, but the treatment of rigid bodies is limited to plane motion with the exception of an introduction to three-dimensional rigid body dynamics as applied to gyroscopes and other axially symmetric bodies.

The general philosophy is that the three-dimensional analysis is given first if it does not add significantly to the complexity of the analysis. The two-dimensional situation then becomes a special case of the general theory. However, if the three-dimen-

sional solution is significantly more complex, the two-dimensional case alone is presented. As an example, cylindrical coordinates present no particular difficulty over polar coordinates, and, thus, the three-dimensional situation is presented. Spherical coordinates, however, are considerably more complex and are omitted.

Limiting the rigid body treatment to plane motion and very simple axially symmetric three-dimensional motion may not appeal to those instructors dedicated to engineering science. However, from a practical standpoint, the amount of three-dimensional analysis that is actually used in industry is very small.

PROBLEM AREAS IN DYNAMICS

One area in which students experience considerable difficulty is in the use of dynamic equilibrium, commonly called d'Alembert's Principle. (Rosenberg, 1968)¹ The current practice in most dynamics textbooks is to apply Newton's Second Law directly and play down the use of artificial equilibrium. In the opinion of the author, this is the simplest and most logical treatment because of the difficulty students find in differentiating between real and inertia forces. Apparently this treatment is not universal because many transfer students claim that their previous instruction in dynamics has involved extensive use of inertia forces. The result is that inertia forces are often included in the summation of forces which is then equated to the product of the mass and acceleration. The author agrees with Professor Meriam that the reduction of a dynamics to a pseudo-statics problem is mainly of historical interest and although its use simplifies certain rigid body problems, it generates many serious difficulties for the student in the fundamental understanding of dynamics.

Early in the semester students appeared reluctant to accept the idea that the differentiation of a vector must include its change in direction as well as in magnitude. Using complete derivations for those cases in which the direction of the unit vector changes such as cylindrical coordinates and motion in a rotating frame of reference seemed effective in impressing the effect of change in direction on the student.

NEW DIMENSIONS

This paper has summarized some of the problems and ideas of the author in presenting a rigorous course in engineering dynamics at the junior level. Many of these problems encountered in trying

¹See also Reader Comment, Journal of Engineering Education, Vol. 59, No. 1, September 1968, pp. 61-62.

to present the general case and then simplifying it for a specific problem appear in companion courses in mechanics of fluids and strength of materials. The general approach appears extremely neat and orderly to an instructor who has considerable understanding of the subject, much of which was probably acquired from the study of simple situations first; to the student, however, it is all too often overwhelming at the first encounter.

We now appear to be entering a new phase with the introduction of programmed dynamics books such as A Programmed Introduction to Dynamics (Work, 1969). However, the basic content of the book is even more important here since the sequence must be followed much more closely than in a conventional course.

APPENDIX--ENGINEERING 135--DYNAMICS

Kinematics of a Particle: Vector displacement, velocity and acceleration; rectangular coordinates; normal and tangential coordinates; cylindrical coordinates; relative motion of two particles; motion in a translating-rotating coordinate system.

Kinetics of a Particle: Newton's laws of motion; units; free body diagrams; rectilinear motion of single and connected particles; rectangular coordinates; normal and tangential coordinates; cylindrical coordinates; elementary orbital mechanics; introduction to mechanical vibrations; d'Alembert's principle; work of a force; potential functions; kinetic and potential energy; impulse and momentum change; variable mass; impulsive force; conservation of linear momentum; angular impulse and momentum; conservation of angular momentum.

Kinetics of a System of Particles: Equations of motion; center and mass; work and energy; linear impulse and momentum; angular impulse and momentum.

Kinematics of Rigid Bodies: Translation and rotation; general plane motion; instantaneous centers of rotation; general motion.

Kinetics of Rigid Bodies—Plane Motion: Equations of motion; kinetic energy; work-energy relations; linear and angular momentum; conservation of linear and angular momentum.

Kinetics of Rigid Bodies—General Motion: Principal axes and moments of inertia; elementary gyroscopic motion.

EXAMINATIONS

Midterms: 3-50-minute tests, closed book, 3-4 problems per test.

Final: 2-hour examination, closed book, 8-10 problems.

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Work, Clyde E., A Programmed Introduction to Dynamics, McGraw-Hill, 1969.

FLOW THROUGH A CONTRACTION

Merle C. Potter
Michael Chojnowski

Several authors of contemporary fluid mechanics texts have incorrectly presented the problem of a parabolic flow undergoing a contraction. The solution to this problem should be of interest to teachers of elementary fluid mechanics as well as to authors who may repeat the error. Actually, the solution presented can be used for fluid entering a contraction with any given velocity profile immediately prior to the contraction.

Consider the flow through the contraction to be steady, two dimensional¹ and incompressible, with zero velocity components perpendicular to the flow at the entrance and exit of the contraction. Losses will be neglected through the short streamlined contraction, and hence, viscosity will have little effect upon the flow and can be neglected. To determine the nature of the velocity distribution at section 2 consider the Euler equations for two-dimensional, steady, inviscid flows

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} \quad (1)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y} \quad (2)$$

and the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

Differentiating Eq. 1 with respect to y and Eq. 2 with respect to x , subtracting results, and with the use of Eq. 3, we have

$$(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}) (\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) = 0$$

¹The problem will be developed for channel flow, and the results for pipe flow presented at the end.

or alternatively,

$$\frac{D\zeta}{Dt} = 0 \quad (4)$$

where ζ is the vorticity and $\frac{D}{Dt}$ is the material (total, substantial co-moving) derivative. According to Eq. 4, the vorticity of a fluid element must remain constant as the element moves through the contraction. This conservation of vorticity requires that the slope of the velocity distribution at section 2 must be the same as the slope for corresponding points (points which lie on the same streamline) at section 1, that is,

$$\frac{du_1}{dy} = \frac{du_2}{d\eta} \quad (5)$$

where y and η are the distances to the same streamline as measured from the walls at sections 1 and 2 respectively. In order that continuity can be satisfied, allow a slip velocity U_o at the boundary, as illustrated in figure 1 (for a viscous fluid there would be a thin boundary layer). One must go downstream sufficiently far for a parabolic profile to again be established; for that case, viscosity must be included, and vorticity would not be conserved.

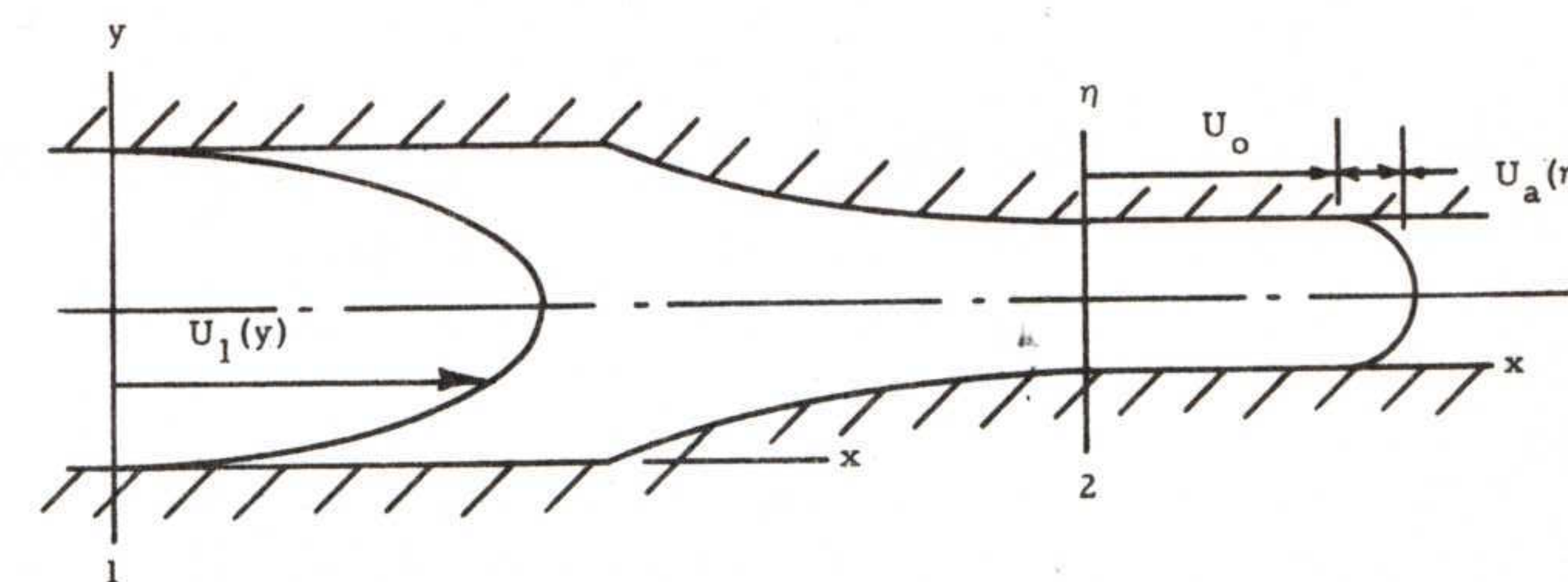


Fig. 1. Flow through a Contraction

Now let us solve for the velocity distribution at Section 2. Continuity between any two neighboring streamlines can be expressed as

$$u_1(y) dy = [U_o + U_a(\eta)] d\eta \quad (6)$$

where U_o is the slip velocity after the contraction and $(U_o + U_a)$ is the velocity at section 2. Eq. 5 is then

$$\frac{dU_1}{dy} = \frac{dU_a}{d\eta} \quad (7)$$

which, when used with Eq. 6 yields

$$U_1 \frac{dU_1}{dU_a} = U_o + U_a \quad (8)$$

After integration, the quadratic expression in U_a is solved to give

$$U_2 = U_a + U_o = \sqrt{U_o^2 + U_1^2} \quad (9)$$

Remember, the above development is for the same streamline. Using Bernoulli's equation along this streamline

$$\frac{U_1^2}{2g} + \frac{P_1}{\gamma} = \frac{U_2^2}{2g} + \frac{P_2}{\gamma} \quad (10)$$

and, substituting for U_2 using Eq. 9, there results the interesting relationship

$$\Delta p = \frac{\gamma U_o^2}{2g} \quad (11)$$

Thus, if the slip velocity after a contraction is known, the pressure drop can be calculated.

To solve for the position at section 2 of a given streamline at section 1, substitute Eq. 9 into Eq. 6 and integrate; then

$$\eta = \int_0^y \frac{U_1}{\sqrt{U_o^2 + U_1^2}} dy \quad (12)$$

Hence, for a given velocity profile $U_1(y)$ at section 1, we can calculate at section 2 the slip velocity from Eq. 12, the total velocity from Eq. 9, and the pressure drop from Eq. 11 for any given contraction.

For a linear velocity profile upstream, the above analysis yields a linear distribution with a slip velocity downstream, which is necessary if vorticity is to be conserved. For the parabolic profile, the integral in Eq. 12 may be evaluated numerically. The downstream velocity distribution for half the channel is shown in figure 2 for a 2 to 1 reduction in crosswise dimension. As the downstream dimension is reduced, the velocity profile becomes more uniform with the slip velocity increasing in magnitude.

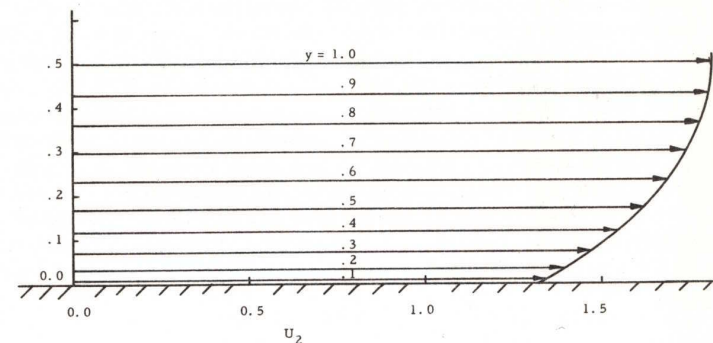


Fig. 1. Velocity Profile for a 2 to 1 Contraction with a Parabolic Profile Upstream

For a circular pipe, the development similar to the above analysis in polar coordinates yields

$$\frac{D}{Dt} \frac{\zeta}{r} = 0 \quad (13)$$

and, thus, the ratio of the vorticity to the radius is conserved as a particle of liquid moves through the contraction. The pressure drop relationship is again

$$\Delta p = \frac{\gamma U_o^2}{2g} \quad (14)$$

The expression locating the downstream streamline is

$$\eta^2 = 2 \int_0^r \frac{U_1 r}{\sqrt{U_0^2 + U_1^2}} dr \quad (15)$$

where η and r are measured from the centerline of the pipe. For a parabolic profile at section 1, this can be integrated, and after some manipulation it can be shown that

$$\frac{U_2}{U_{\max}} = \frac{1}{2} \left[\frac{R_1^2}{R_2^2} + \frac{R_2^2}{R_1^2} \right] - \frac{\eta^2}{R_1^2} \quad (16)$$

where $U_{\max}$ is the velocity at the centerline upstream and R_1 and R_2 are the radii of the pipe.

Again, viscous effects were neglected in the short contraction. Nothing has been assumed about how the parabolic profile is established prior to the contraction; certainly viscous effects in a long reach of pipe or channel could accomplish this.

It is obvious why a contraction in a wind tunnel creates essentially uniform flow in the test section. Variations in the velocity profile prior to the contraction are hardly noticed downstream.

FUNCTIONAL CHARACTERISTICS OF FLUIDIC ELEMENTS

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Engineering Manager
Medlab Computer Services, Inc.

TERMINOLOGY

Several terms are used to describe the characteristics of fluidic devices which may or may not be familiar to those first investigating fluidics. In order to provide a common understanding of these terms, a brief discussion of each is provided here.

Digital/Proportional--Digital devices are those devices which have discrete levels of output. By far the most common digital devices are binary devices having two discrete levels--on or off. Proportional devices are those devices which have variable output levels which are proportional to the input signal. The output level is a multiple of the input level.

Pressure and Flow Range--There are three general levels at which fluidic signals can be classified. The lowest level is that level of signals associated with sensors. The pressures of these low level sensor signals are between a fraction of an inch of water column to a few pounds per square inch (PSI). Flows at the low levels are generally from .001 SCFM to 1 SCFM (standard cubic feet per minute). The medium range pressures are from about $\frac{1}{2}$ psi to about 30 psi. The associated flows are from about .01 SCFM to about 5 SCFM. Medium level signals are commonly those found on the inputs and outputs of many fluidic logic functions. The upper range is associated with power devices such as cylinders and air motors. The pressures at the upper range vary between 5 psi and 125 psi and the flows are from 1 SCFM to 20 SCFM. Fluidic devices are available in all three ranges. The choice of which device to use is dependent on several factors including air consumption, number of devices required, time response, and, of course, the application.

Gain--Gain is the ratio between the change in output pressure (or flow) to the change in input pressure (or flow). For example, if an output pressure can be changed from 10 psi to 0 psi with a control signal change from 0 psi to 5 psi, the gain is -2 (the negative sign indicates that inversion of the signal occurred).

Frequency response/time response--Frequency response refers to the range of frequencies through which a device can respond acceptably. With proportional devices, the gain of a device will decrease as the frequency of the input is varied above and below the normal operating range of the device. With digital devices, the ability to switch according to an input frequency will decrease as the frequency of the input is varied

beyond the normal operating range. Time response is the amount of time that passes between the time an input changes by 90% of the maximum input pressure (or flow) and the time the output pressure changes by 90% of maximum output pressure.

Fan-in/Fan-out--Fan-in refers to the number of independent inputs which can be applied to a device. Generally, this is the number of input channels available. Fan-out refers to the number of similar fluidic devices which can be driven reliably from one output signal. This is a crude but useful measure of the output impedance of a device.

Load Sensitivity--The load of a device is the amount of flow that is required by another device being driven by the first device under measurement. Some fluidic devices can be sensitive to varying loads and can in fact become quite unstable with large load variations, if the circuits are not properly designed.

Pressure recovery/flow recovery--The pressure at the output of a device is lower than the supply pressure. The amount of output pressure, expressed as a percentage of the supply, is called the pressure recovery. Flow recovery is the percentage of supply flow which is recovered at the output.

Active and Passive--Active devices are those devices which must have a separate supply applied to the device in addition to the input and output connections. Active devices are also noted for having a gain greater than one, that is, the output signal is a multiple of something greater than one times the input signal. Passive devices are those which require no supply and have gains of less than one.

WALL ATTACHMENT

Fluidic devices which use the wall attachment or coanda effect are the most widely known of the fluidic family. This is due in part to the fact that the first major attempt at using fluids, and particularly their dynamic properties, to perform logic or amplification was done with all attachment devices. Wall attachment can occur effectively with any fluid although air is the most commonly used. When a jet of relatively high velocity fluid enters a region containing a fluid, the jet will entrain some of the fluid of the region. If a space near the jet can be confined such as by placing a wall near the jet, a significant pressure differential can be developed in the confined space. If the pressure in the confined space is lower than the pressure on the opposite side of the jet, the jet will bend toward the lower pressure space. This jet can in fact be bent far enough to actually flow against the wall or in effect attach itself to the wall. A recovery channel can be placed near the wall through which the attached jet can be recovered for further use. Figure 1 shows the wall attachment effect.

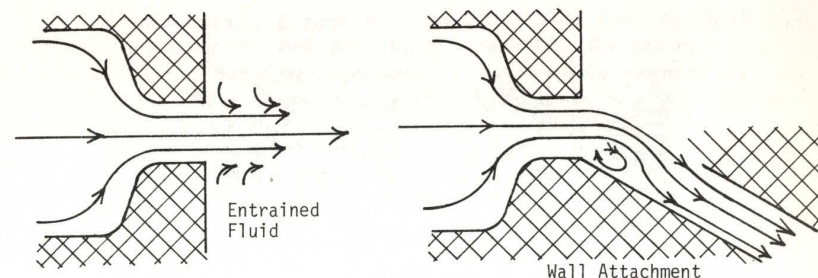


Fig. 1.

The jet, which has been attached to a wall, can be moved away from the wall by increasing the pressure in the low pressure space until the increasing pressure is greater than the pressure on the opposite side of the wall. The increasing pressure can be provided through an input channel to the interaction region. If a second wall and associated input channel are located near the jet, the jet can be switched from one wall to the other and once switched, the jet will remain on the wall to which it was attached even though the input signal has been removed. This, in its simplest form, is a bistable device whose two stable states are first when the jet is attached to the first wall and the second when the jet is attached to the second wall. Figure 2 shows the bistable device with the proper recovery or output channels also included. Also, note the splitter which is located symmetrically between the two output channels so that neither channel is favored by the jet.

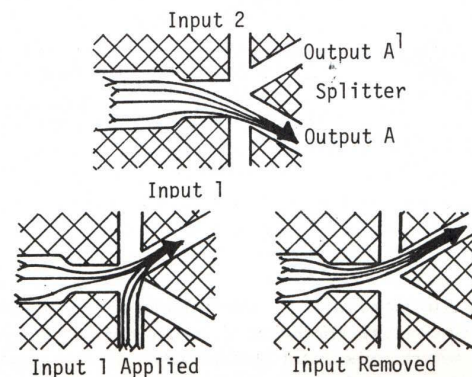


Fig. 2

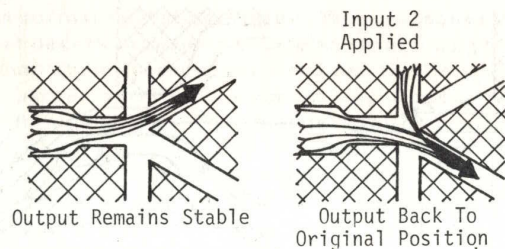


Fig. 2 Continued

A second wall attachment device is an OR/NOR gate. Its function is such that when no input is present the jet is always flowing out a predetermined channel regardless of the previous input signals. The jet can only be switched to the second channel when an input signal is present. The actual construction of the OR/NOR gate is very similar to the bistable except for either or both of two variations.

The first is to provide a channel on one side of the jet which is much larger than the input channel on the opposite side. This new channel, called a vent, must be large enough to provide plenty of flow into the interaction space so that no low pressure space is developed on the vent side when no input signal is present in the opposite channel. The second variation is to locate the splitter off center of the axis of the jet so that one output channel is favored over the other. Figure 3 shows an OR/NOR gate with two input channels on one side opposite the vent.

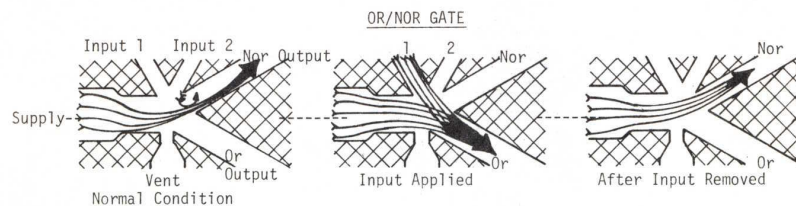


Fig. 3

The output flow always exists out the NOR leg when neither input A nor input B is present. However, if either input A or input B is present, the output exists out the OR leg.

Obviously, wall attachment is not needed to produce the OR/NOR effect. Wall attachment is used to insure a more near zero pressure out the unused output leg and also to provide a little better gain.

Since any logic can be done with a NOR gate, the NOR device is the backbone of any logic capability. OR gates, bi-stables, ANDS, NANDS, One Shots, etc. can all be made using NOR gates. An OR function, as shown in Figure 4, can be made using two NOR gates.

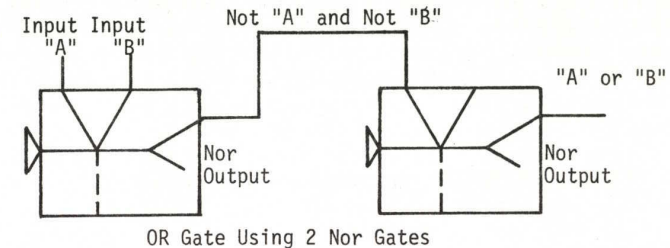


Fig. 4

As will be apparent later, not all NOR gates have also the OR output. This, then, is one advantage of the wall attachment OR/NOR gate when number of gates (and therefore size and cost) is important.

The bistable device is useful to the wall attachment family because the memory function can be accomplished with one device instead of two as would be required if only NOR functions were available.

Wall attachment devices have gained wide acceptance because of the wide range of supply pressures at which they can be operated. Normally, this range is from about $\frac{1}{4}$ psi to 30 psi. These devices are advantageous because they can provide the signals to drive many low power to medium power cylinders, valves, motors, etc. The supply flow required is from about .1 SCFM to 1 SCFM, which means again the flow necessary to drive low power to medium power devices is available.

However, this flow requirement also leads to one of the major disadvantages of wall attachment devices and that is the problem of air consumption at the logic level. A 100 gate system can require from 15 to 50 SCFM of air which generally is a strain on a user's air system. The pressure recovered with wall attachment devices is about 30% which is among the lowest of all the fluidic effects, but is not generally a serious disadvantage. The gain range of wall attachment devices is from 1 to 10 and generally is about 3. The time response of wall attachment devices is among the best of the fluidic devices and is in the order of 1 millisecond with the practical time being between 2 and 3 milliseconds. This time response is then one of the major advantages

of wall attachment. The Fan-In or number of possible inputs is limited in most cases to two inputs; however, a few devices can be found with four inputs. The Fan-Out of wall attachment devices is limited to about 10 devices, which is very high for fluidic devices and exists because of the high flow consumption of the device. The load sensitivity of wall attachment devices has been one of the major disadvantages and is the cause of much of the difficulty in engineering fluidic circuits using wall attachment devices. The best solution to the load sensitivity of the devices has been to add vents on the output legs of the gate so that as the load decreases the flow, which is not required, is diverted out the vent instead of upsetting the wall attachment.

Typically, the channels of a wall attachment device have areas of between 4×10^{-4} to 8×10^{-8} square inches. The channel areas must be kept small to keep air consumption down. On the other hand, the smaller the channels, the worse the problem of air contamination control.

To recap, the advantages of wall attachment devices are:

1. Fast time response
2. Ability to provide low or medium power signals
3. Wide range of supply pressures

The disadvantages in comparison to other fluidic devices are that they require extra circuit designing to overcome load sensitivity, require higher air consumption, and require more air contamination control than most fluidic devices.

JET INTERACTION

A jet stream can be diverted or caused to change direction by another jet positioned at some angle from the main jet stream. The output pressure and flow which is actually recovered is then a function of the amount of the supply jet, directed into the output channel. Figure 5 shows one form of a Jet Interaction device.

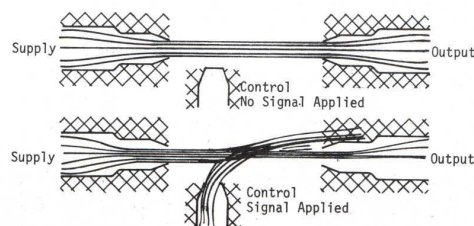


Fig. 5

Generally, these devices are proportional in nature, that is, the output level of pressure can be varied to any desired level by applying the proper input level. It is also possible to have two output channels which give, in effect, a pushpull signal. Figure 6 shows this device.

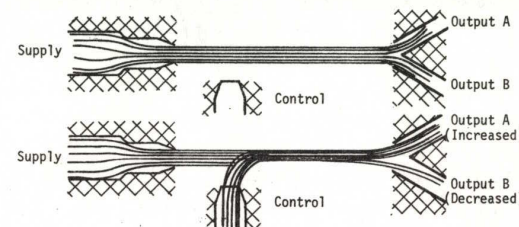


Fig. 6

The pressure and flow ranges of these devices can be found in both the low level and the medium level ranges. Typically, the pressures are about .1 to 10 psi. Because jet interaction devices are active, they have gain which is usually between 1 and 10. By cascading several amplifiers the total gain can be increased.

The frequency response of these devices can be quite high approaching a few thousand cycles per second. The pressure recovery of jet interaction devices is in the order of 30% which is similar to the wall attachment devices.

One device which falls in the general classification of jet interaction devices is called the Momentum Amplifier. It is named so because the pressure gain and pressure recovery is a function of the momentum of the supply jet and control jet. The Momentum Amplifier is unusual because of the higher gain available which can be in the order of 30. Also the pressure recovery is higher than the normal.

TURBULENCE AMPLIFIER

The pressure that can be recovered from a laminar jet is higher than that recovered from a turbulent jet of the same source. This property of jets can be utilized to produce an active NOR gate since a laminar jet can be changed to a turbulent jet by applying another jet of fluid at an angle to the main jet similar to that in a jet interaction device. Figure 7 shows a Turbulence Amplifier. The laminar jet is established by passing flow through a long tube. As yet, no commercially successful Turbulence Amplifier has been designed that works with liquids.

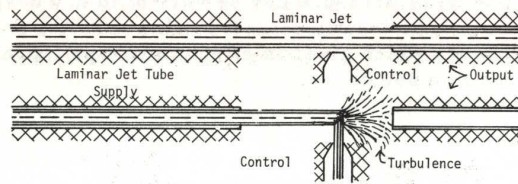


Fig. 7

The pressure range of Turbulence Amplifiers is between a fraction of an inch of water column to about 1 psi. The flow range is very low, typically .01 SCFM for output signals.

The pressure gain can be fairly high in the order of 20-30. The pressure recovery is somewhat higher than wall attachment devices and can be as high as 60%. The fan-out of T.A.'s (Turbulence Amplifiers) vary between about 4 and 8 while the usual fan-in is 4. Turbulence Amplifiers are quite load insensitive and can be operated from dead-end (no flow) to full flow. The time response of Turbulence Amplifiers is lower than wall attachment and is about 3 milliseconds.

One unusual property of Turbulence Amplifiers is that there are several ways of causing a jet to change from laminar to turbulent. The switching can be caused by electric sparks, heat, acoustic signals, other jets or even breezes. These different modes of switching make the Turbulence Amplifiers ideal for various sensors but at the same time, leads to its major disadvantage which is that of being switched by doors slamming, high frequency acoustic noises, etc.

IMPACT MODULATOR

This device could be classified with jet interaction devices, but because it has a unique way of recovering the desired signal, it is considered separately here. In an impact modulator device, two jets are pointed toward each other along the same axis so that their two jets impact on each other. At the impact point flow exists at right angles to the axis of the jets; this flow can be recovered to produce either a proportional device or a digital device.

The input signal is a jet applied such that it interferes with the supply by decreasing its momentum and, therefore, shifting the point of impact closer to the main supply. Figure 8 shows a diagram of the impact modulator. The characteristics of this device such as pressure range, etc. are similar to those of jet interaction devices.

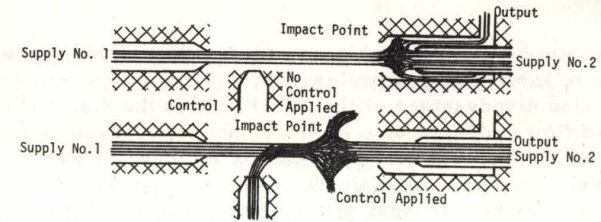


Fig. 8

WAVE PHENOMENA CONTROL

Wave phenomena control is the latest of the various fluidic effects which has been put to use in the design of fluidic devices. When a jet issues from a nozzle of a specific design and through a range of pressures which is related to the nozzle size, a diamond pattern of flow will develop. This supply jet is also collected in a volume. As the pressure in the volume approaches the value of pressure at the entrance, the flow is diverted away from the main axis similar to the same patterns as in an impact modulator. The point of flow diversion will switch back along the diamond pattern toward the supply jet and the fluid which has filled the volume will flow back out toward the supply nozzle since the pressure at entrance has decreased. The pressure in the volume can decrease to a value slightly less than ambient. The pressure variation can be quite large approaching a 50 psi change. It is this large pressure variation that provides this device with its main advantages. Once the volume pressure decreases below a critical point, the supply jet pattern will revert to the original pattern and repeat the process of charging the volume and discharging. Figure 9 shows the basic wave phenomena control device.

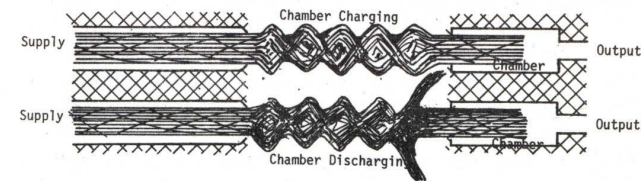


Fig. 9

The device, which is called a HPLF and means High Pressure Low Flow, is useful as an oscillator or timer. A digital amplifier has also been developed using this principle. Development of OR, NOR, etc. are progressing so that they will also be available. The pressure range of this device is at the bottom end of the high range, that is, about 30 to 50 psi. The pressure recovery approaches 90%, one of the highest. Flows are from 0.1 SCFM to 1.5 SCFM and since the effect is dependent on flow,

the output must be a low requirement. In other words, it is load sensitive to large changes in flow.

One disadvantage of the HPLF is that the diamond pattern of flow produced also produces acoustic waves, meaning the device is somewhat noisy and can be a concern if very low noise levels are to be maintained.

Vortexing--Vortexing is an effect whereby the output flow rotates about the axis of output port. The greater the rotation of flow, the greater the restriction to flow, and, therefore, the less the actual output flow. With no control signal applied the flow path is directly from a supply port to the output port of the circular chamber. If a control signal is applied tangentially to the chamber, it will cause the supply flow to rotate around in the chamber before exiting. Increasing the control signal will cause the rotation to increase and the output flow to decrease. The device, then, is proportional. Figure 10 shows a vortex amplifier.

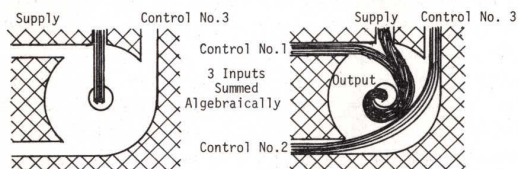


Fig. 10

One advantage of the vortex amplifier is that several inputs can be added so that each input level is added to affect the output. Also the inputs can be aligned so that they oppose each other. The result is an excellent summing junction whereby the output is the algebraic sum of the inputs.

The vortex amplifier is a good flow modulator, particularly at the high pressure and flow range, and can be used to control air motors, cylinders, etc. One disadvantage of the vortex amplifier is that the control pressure must be greater than the supply pressure. The supply flow is larger than the control pressure and, therefore, the vortex amplifier should be used where flow modulation is needed, but when pressure gain is not needed.

CONCLUSIONS

This paper has given a brief discussion of the characteristics of fluidic devices as they apply to circuits as well as a brief description of the functional characteristics which make the devices work. It is hoped that the observer will be able to better analyze fluidics in the many applications which are possible.

VELOCITIES AND ACCELERATION IN CYLINDRICAL AND SPHERICAL COORDINATES

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Almost every textbook on elementary dynamics which covers motion in three dimensions, also includes under the subject of particle Kinematics a list of representations of velocity and acceleration in cylindrical and spherical coordinates; see, for example, References on page 24. In those cases where the representations are developed, it is essentially in terms of a pictorial visualization of the effect of a change in the position vector upon the changes (rotations) of the orthonormal set of unit vectors, $(\hat{e}_r, \hat{e}_\phi, \hat{e}_z)$ for cylindrical coordinates, Fig. 1, and $(\hat{e}_r, \hat{e}_\theta, \hat{e}_\phi)$ for spherical coordinates, Fig. 2. The procedure is then to express the time derivative of position (or velocity) as the sum of the products of the unit vectors and the components, and to take the derivative to obtain the velocity (acceleration) where both the component and the unit vector will experience changes.

The procedure requires a great number of substitutions, and a great deal of accounting skill to keep track of what changes with what. It does have the advantage of requiring little mathematics, and is pedagogically probably justified for the somewhat simpler case of cylindrical coordinates to help the student visualize just what these changes are. The procedure is so cumbersome in the case of spherical coordinates, however, that little insight is gained from it. The writer has for some time now used an alternative procedure which has the advantage of applying a fundamental relation for vectors in moving frames that usually precedes the coordinate transformations in most texts, and is mathematically more straightforward. In the classroom, to get the greatest benefit from both presentations, the cylindrical coordinates are first handled by the text method, and then by the new method presented below. The results are compared as identical, and spherical coordinates are then handled by the new method alone.

The method is based on two concepts: 1. cylindrical and spherical coordinate systems are both rotating reference frames; 2. the well-known relationship that the time derivative of a vector which is referred to a moving coordinate system (i. e., whose components are expressed in terms of the unit vectors of the moving coordinate system) is given by:

$$\frac{d\bar{A}}{dt} = \frac{\partial \bar{A}}{\partial t} + \bar{\omega}_f \times \bar{A}$$

where $\bar{A}$ is the vector, expressed in components along the moving coordinate axes; $\bar{\omega}_f$ is the angular velocity of the coordinate system similarly expressed; $\frac{\partial}{\partial t}$ represents the time derivative of the individual component of the vector as represented in the moving coordinate system, i.e., as would be seen by an observer fixed in and moving with that coordinate system. It should be noted that $\frac{d\bar{A}}{dt}$ is the absolute rate of change of the vector $\bar{A}$ but it too is referred to, or "expressed in terms of," the moving coordinate system.

For cylindrical coordinates, the position vector $\bar{r}$ is given by

$$\bar{r} = r_0 \hat{e}_r + 0 \hat{e}_\phi + z \hat{e}_z$$

or simply as $(r_0, 0, z)$ in terms of its components. The frame rotates with angular velocity $\dot{\phi}$ about the z axis ($\hat{k}$ in the Cartesian system, $\hat{e}_z$ in the cylindrical), so that

$$\bar{\omega}_f = \dot{\phi} \hat{e}_z = (0, 0, \dot{\phi})$$

The velocity may then be obtained from

$$\begin{aligned} \bar{v} &= \frac{d\bar{r}}{dt} = \frac{\partial \bar{r}}{\partial t} + \bar{\omega}_f \times \bar{r} \\ &= \frac{\partial}{\partial t} \begin{bmatrix} r_0 \\ 0 \\ z \end{bmatrix} + \begin{bmatrix} \hat{e}_r & \hat{e}_\phi & \hat{e}_z \\ 0 & 0 & \dot{\phi} \\ r_0 & 0 & z \end{bmatrix} = \begin{bmatrix} \dot{r}_0 \\ 0 \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\phi} r_0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{r}_0 \\ \dot{\phi} r_0 \\ \dot{z} \end{bmatrix} \\ &= \dot{r}_0 \hat{e}_r + \dot{\phi} r_0 \hat{e}_\phi + \dot{z} \hat{e}_z \end{aligned}$$

exactly as given in the texts. The dot over a term has the same meaning as $\frac{\partial}{\partial t}$, and the row or column vector has been used interchangeably as convenient, since no confusion results. Similarly, the acceleration is obtained from

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{\partial \bar{v}}{\partial t} + \bar{\omega}_f \times \bar{v}$$

$$\begin{aligned} &= \frac{\partial}{\partial t} \begin{bmatrix} \dot{r}_0 \\ \dot{\phi} r_0 \\ \dot{z} \end{bmatrix} + \begin{bmatrix} \hat{e}_r & \hat{e}_\phi & \hat{e}_z \\ 0 & 0 & \dot{\phi} \\ \dot{r}_0 & \dot{\phi} r_0 & \dot{z} \end{bmatrix} = \begin{bmatrix} \ddot{r}_0 \\ \ddot{\phi} r_0 + \dot{\phi} \dot{r}_0 \\ \ddot{z} \end{bmatrix} + \begin{bmatrix} -\dot{\phi}^2 r_0 \\ \dot{\phi} \dot{r}_0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \ddot{r}_0 - \dot{\phi}^2 r_0 \\ 2\dot{\phi} \dot{r}_0 + \ddot{\phi} r_0 \\ \ddot{z} \end{bmatrix} \\ &= (\ddot{r}_0 - \dot{\phi}^2 r_0) \hat{e}_r + (\ddot{\phi} r_0 + 2\dot{\phi} \dot{r}_0) \hat{e}_\phi + \ddot{z} \hat{e}_z \end{aligned}$$

again in agreement with the texts.

For spherical coordinates, the position vector is given by

$$\bar{r} = r \hat{e}_r = (r, 0, 0)$$

and the angular velocity results from the vector sum of the angular velocities due to the change of ϕ , $\dot{\phi} \hat{k}$, and due to the change in θ , $\dot{\theta} \hat{e}_\phi$. Since the unit vector $\hat{k}$ is given in the spherical coordinates by

$$\hat{k} = \cos \theta \hat{e}_r - \sin \theta \hat{e}_\theta$$

the sum is

$$\bar{\omega}_f = \dot{\phi} \cos \theta \hat{e}_r - \dot{\phi} \sin \theta \hat{e}_\theta + \dot{\theta} \hat{e}_\phi = (\dot{\phi} \cos \theta, -\dot{\phi} \sin \theta, \dot{\theta})$$

We thus have for the velocity

$$\begin{aligned} \bar{v} &= \frac{d\bar{r}}{dt} = \frac{\partial \bar{r}}{\partial t} + \bar{\omega}_f \times \bar{r} \\ &= \frac{\partial}{\partial t} \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{e}_r & \hat{e}_\theta & \hat{e}_\phi \\ \dot{\phi} \cos \theta & -\dot{\phi} \sin \theta & \dot{\theta} \\ r & 0 & 0 \end{bmatrix} = \begin{bmatrix} \dot{r} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\theta} r \\ \dot{\phi} r \sin \theta \end{bmatrix} \\ &= \begin{bmatrix} \dot{r} \\ \dot{\theta} r \\ \dot{\phi} r \sin \theta \end{bmatrix} \\ &= \dot{r} \hat{e}_r + \dot{\theta} r \hat{e}_\theta + \dot{\phi} r \sin \theta \hat{e}_\phi \end{aligned}$$

and for acceleration

$$\begin{aligned}
 \bar{a} &= \frac{d\bar{v}}{dt} = \frac{\partial \bar{v}}{\partial t} + \bar{\omega}_f \times \bar{v} \\
 &= \frac{\partial}{\partial t} \begin{bmatrix} \dot{r} \\ \dot{\theta} r \\ \dot{\phi} r \sin \theta \end{bmatrix} + \begin{bmatrix} \hat{e}_r & \hat{e}_\theta & \hat{e}_\phi \\ \dot{\phi} \cos \theta & -\dot{\phi} \sin \theta & \dot{\theta} \\ \dot{r} & \dot{\theta} r & \dot{\phi} r \sin \theta \end{bmatrix} \\
 &= \begin{bmatrix} \ddot{r} \\ \ddot{\theta} r + \dot{\theta} \dot{r} \\ \ddot{\phi} r \cos \theta + \dot{\phi} \dot{r} \sin \theta + \ddot{\phi} r \sin \theta \end{bmatrix} + \begin{bmatrix} -\dot{\phi}^2 r \sin^2 \theta - \dot{\theta}^2 r \\ \dot{\theta} \dot{r} - \dot{\phi}^2 r \cos \theta \sin \theta \\ \dot{\phi} \dot{\theta} r \cos \theta + \dot{\phi} \dot{r} \sin \theta \end{bmatrix} \\
 &= \begin{bmatrix} \ddot{r} - \dot{\phi}^2 r \sin^2 \theta - \dot{\theta}^2 r \\ 2\dot{\theta} \dot{r} + \ddot{\theta} r - \dot{\phi}^2 r \cos \theta \sin \theta \\ 2\dot{\theta} \dot{\phi} r \cos \theta + 2\dot{\phi} \dot{r} \sin \theta + \ddot{\phi} r \sin \theta \end{bmatrix} \\
 &= (\ddot{r} - \dot{\phi}^2 r \sin^2 \theta - \dot{\theta}^2 r) \hat{e}_r + (2\dot{\theta} \dot{r} + \ddot{\theta} r - \dot{\phi}^2 r \cos \theta \sin \theta) \hat{e}_\theta \\
 &\quad + (2\dot{\theta} \dot{\phi} r \cos \theta + 2\dot{\phi} \dot{r} \sin \theta + \ddot{\phi} r \sin \theta) \hat{e}_\phi
 \end{aligned}$$

both of which agree with the textbooks.

A further alternative is the use of Lagrange's equations to carry out the transformation as is done in Reference 3 for spherical coordinates, but particle kinematics is usually covered before Lagrangian methods are introduced, and these methods are considerably more abstract than that given above.

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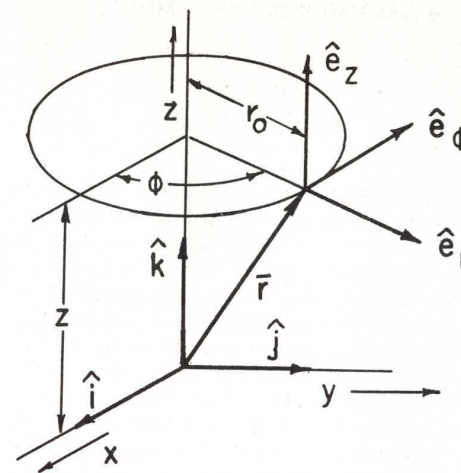


Fig. 1. Cylindrical coordinate system

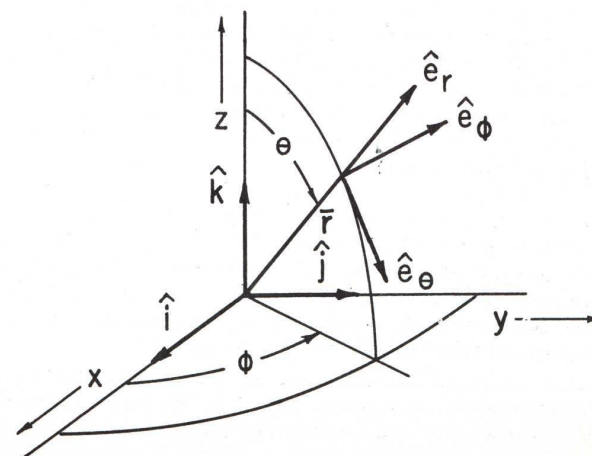


Fig. 2. Spherical coordinate system

DEVICE TO MEASURE RIGID DISPLACEMENTS USING MOIRE

A. J. Durelli
V. J. Parks

Moire has become a popular method to measure displacements. Of all experimental stress analysis methods, it is the one on which the most research is being conducted at present, partly because moire is the most recent method found to measure displacements, but also because moire is an extremely versatile method. Educational courses in physics, mechanics, and stress analysis have introduced much material on moire, but until now, the practical aspects have been difficult to transmit to students, mainly because of the relatively large expense involved in obtaining gratings. Important improvements have been made recently in that respect; inexpensive coarse gratings are available from drafting supply houses,¹ and large, dense gratings of good quality are also available commercially.²

It has been shown how precise structural analysis can be conducted using coarse gratings (Durelli and Daniel, 1962), and the method has been found very useful for teaching purposes. However, no simple device has yet been prepared to show the application of moire to the measurement of rigid motions, translations, or rotations. Such a device is described below.

The board shown in figure 1 can be made using any easily available plastic sheet, usually plexiglas. A horizontal grating, with a pitch of about 60 lines per inch, is cemented to the top surface of the board. Two parallel strips of plexiglas, one inch wide, and about one-fourth inch thick, are screwed to the board about an inch apart. Similarly, two strips, parallel to each other but inclined with respect to the axis of the board, are screwed on the board. Blocks can slide between these strips. On the bottom surface of two such blocks, gratings with lines parallel to the board gratings are cemented. On the bottom surfaces of two other blocks, gratings that have some inclination with respect to the master gratings are cemented to produce an interference pattern of fringes, designated "mismatch" in the figure.

¹One manufacturer is Para-tone, Inc., 512 Burlington Ave., Lagrange, Ill. The sheets are sometimes called Zip-a-tone.

²E. E. Brown & Co., Ltd., Empire Works, Cork Lane, Glen Parva, Leicester, England.

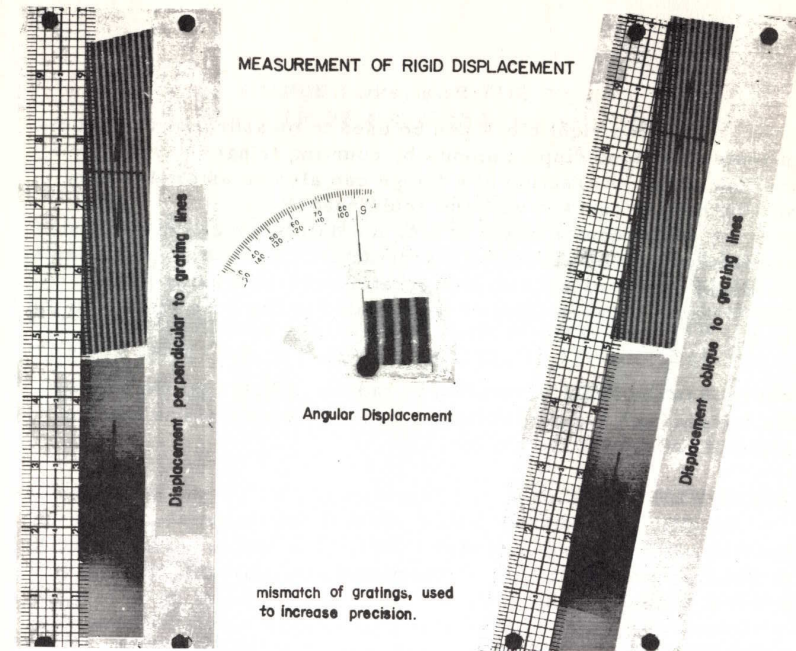


Fig. 1. Translucent Board with Coarse Gratings to Illustrate the Measurement of Rigid Translation and Rotation Using Moire

The left bottom block can be used to measure vertical displacements by counting the alternate cycles of darkness and light in the field. The displacement corresponding to each passage from light to darkness, or darkness to light, is equal to one half the pitch of the grating and can be checked against the scale. When n is the number of cycles, from either dark to dark or light to light, p is the pitch, and v is the vertical displacement, $v = np$.

The right bottom block can be used to measure the vertical component of the displacement along the inclined strips. There the displacement can be measured with the scale, and the vertical component will be given by the cyclic appearance of darkness and light, using the same equation as above. A comparison with the scale can be made if the angle between the strips and the direction of the grating is known, and the vertical component of motion is computed from trigonometry.

The top left block can be used to measure vertical displacements by counting the number of fringes that pass by a point marked on the block. It can be verified that the answer obtained is the same as the one obtained with the left bottom block, but more precise measurement can be made, and interpolation of fractions of a fringe is possible.

The top right block can be used to measure vertical components of oblique displacements by counting fringes passing by a marked point. A fraction of a fringe can also be estimated here, and the motion checked with the scale reading.

In the center of the board, a small block can be rotated, and the rotation measured on a protractor. As with displacements, rotations can be measured with greater precision using moire. The distance between fringes δ is related to the pure rigid rotation

$$\theta \text{ by: } \delta = \frac{\rho}{\theta}$$

where θ is the angle of rotation in radians. Note that the angle of the fringes with the perpendicular to the grating lines is half the angle of rotation of the block.

ACKNOWLEDGEMENT

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A SIMPLE FORM FOR THE LENGTH OF A CATENARY

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One of the approximate formulas relating the quantities of a catenary cable (Fig. 1) is (Synge and Griffith, 1959)

$$a \approx s \left[1 - \left(\frac{2}{3} \right) \left(\frac{h^2}{s^2} \right) \right] \quad (1)$$

where a is one-half the span length, h is the sag, and s is the distance, measured along the curve, from the lowest point to the support. To determine s , if h and a are given, we may use Eq. 1 and solve a quadratic equation in s . The approximate value of s is usually found to be less than the exact one. In this brief note we give a formula that relates the same three quantities but it will be simpler and more accurate than Eq. 1.

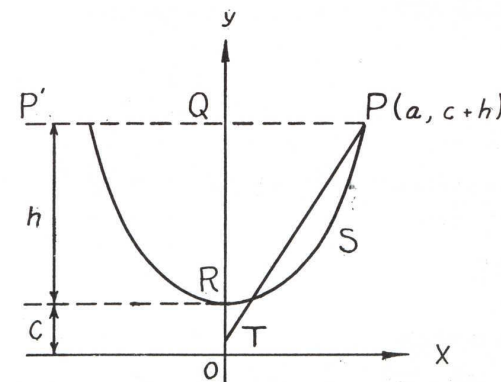


Fig. 1. A Catenary Cable

Corresponding to Fig. 1, the equation of the catenary is

$$y = c \cosh (x/c) \quad (2)$$

where c is called the parameter of the catenary. If the cable is tightly stretched, the value of c is very large. Hence we may write

$$\begin{aligned} s &= \int_0^a \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = c \sinh \left(\frac{a}{c}\right) \\ &= c \left[\left(\frac{a}{c}\right) + \left(\frac{1}{6}\right) \left(\frac{a}{c}\right)^3 + \left(\frac{1}{120}\right) \left(\frac{a}{c}\right)^5 + \dots \right] \\ &\approx a + \frac{a^3}{6c^2} \end{aligned} \quad (3)$$

If P_1 is the support point, so that $x = a$, and $y = c + h$, then Eq. 2 becomes

$$\begin{aligned} c + h &= c \left[1 + \left(\frac{1}{2}\right) \left(\frac{a}{c}\right)^2 + \left(\frac{1}{24}\right) \left(\frac{a}{c}\right)^4 + \dots \right] \\ &\approx c + \left(\frac{1}{2}\right) \left(\frac{a^2}{c}\right) \end{aligned} \quad (4)$$

or

$$h \approx \left(\frac{1}{2}\right) \left(\frac{a^2}{c}\right)$$

On the vertical axis let some point T be chosen such that $\overline{TP} = \overline{RP} = s$,

and assume that

$$\overline{TQ} = k (\overline{RQ})$$

Then from the triangle

$$\overline{TP}^2 + \overline{QP}^2 = \overline{TP}^2, \text{ i.e., } k^2 h^2 + a^2 = s^2 \quad (5)$$

Elimination of s between Eq. 5 and Eq. 3 gives

$$k^2 \approx \frac{a^4}{3c^2 h^2} + \frac{a^6}{36c^4 h^2} \quad (6)$$

The second term of high order on the right side of Eq. 6 may be neglected because in many practical circumstances

$$\left| \frac{a^3}{6c^2 h} \right| < 1$$

Thus Eq. 6 simplifies to

$$k \approx \frac{a^2}{\sqrt{3} ch}$$

and by using Eq. 4 it becomes

$$k \approx \left(\frac{a^2}{\sqrt{3}c} \right) \left(\frac{2c}{a^2} \right) = \frac{2}{\sqrt{3}} \quad (7)$$

Substituting Eq. 7 in Eq. 5 we have

$$s \approx \sqrt{\left(\frac{4}{3}\right) h^2 + a^2} \quad (8)$$

Obviously, the approximate value of k is greater than the true k , so the approximate value of s given by Eq. 8 will always be greater than the true s . To show the accuracy of Eq. 8, let $h = 52.8$ ft. and $a = 80$ ft. The values of s are 98.8 ft. and 100.6 ft. from Eq. 1 and Eq. 8, respectively. Since the true value of s is 100 ft. (Merriam, 1968), the latter represents an error of .6% which is only half as great as that represented by the former.

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THE SOURCE, FUNDING, AND DESTINATION OF* GRADUATE STUDENTS IN MECHANICS

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A history of the source, funding, and destination of graduate students who have completed their degrees would involve a host of variables and a complex study. Data are discontinuous and scrambled enough to support almost any preconceived conclusion. Unscrambling available data and interpreting it is even harder. Rather than bore you with statistics, I prefer to cite prejudices--mine as well as those of a few of my peers. Prediction is difficult enough when it deals only with the future and when all of the variables involved cover the entire time span being considered.

Such is not the case here. Statistics covering these enrollments vary as departments are merged or splintered, and local enrollments thus appear to be discontinuous. Graduate students majoring in mechanics do so in separate mechanics departments, if these exist at a particular school; in traditional engineering curricular departments, whether or not separate mechanics departments do exist at that school; or in more "modern" engineering science departments. Whether to include engineering physics, as is now being done in Engineering Manpower Commission statistics, is questionable.

Before 1964, when separate statistics were being reported for mechanics majors by the Department of Health, Education, and Welfare and the American Society for Engineering Education (ASEE), more than two M.S. and one Ph.D. degrees were awarded for every B.S. degree. Not all B.S. graduates went on to graduate, although most of them did. More B.S. mechanics majors of my acquaintance now choose industry over graduate school; hence, many students have had to transfer into mechanics at the graduate level to make up this deficit.

FACTORS AFFECTING ENROLLMENT

The number of students transferring from traditional departments has decreased as these departments developed programs emphasizing their engineering science to a more marked degree. Graduate study in traditional curricular departments has tended to become more scientifically applied rather than design-

oriented, as new faculty members were appointed with little or no practical engineering experience.

During the past 20 years, graduate students majoring in mechanics, either in separate mechanics departments or in traditional engineering curricular departments, found a ready market and multiple job offers in industries and in universities. Almost unlimited financial support was available from the federal government or educational foundations or from industries, especially when they were engaged in federally sponsored contracts. The majority of graduates with whom I am acquainted preferred academic appointments, not only because these graduates prefer teaching, but because, if they were not U.S. citizens and hoped to remain in this country, they created an unacceptable security risk in defense-oriented industries. During 1969, the job situation has been completely reversed. Academic staff vacancies have shrunk to virtually zero, and openings in industry for mechanics graduates have been curtailed to a point where only one or two offers were received per student on the B.S., M.S., or Ph.D. levels. Industry was quite willing to employ engineers on R&D assignments as long as the government was paying for the work and the companies earned 100 to 200% overhead. The cut in these federal funds signaled an end to the production of such projects, particularly those that were analytical in nature and produced only software whose ultimate destination was a filing cabinet or a publication.

Likewise, the change in the draft law, from the deferred shelter of the graduate sanctuary to elimination of all critically skilled deferments, has reoriented the desires of B.S. graduates toward full-time graduate status.

DECREASING ENGINEERING ENROLLMENTS

Thus, the requirements of industry and of the armed forces contributed to a decrease in enrollment in engineering in general for the 1969-70 academic year of 18% for M.S. and 9% for Ph.D. full-time students (NSPE). "Since the elimination of draft deferments two years ago, engineering master's enrollments have fallen from a high of 34,000 in the fall of 1967 to 20,000." Enrollment of foreign graduate students increased markedly and made up one-fifth of this loss. Part-time enrollment increased, however, as students elected to continue graduate study while being employed full-time in industry. Undergraduate enrollments decreased noticeably in the freshman and sophomore years (NSPE, 1970).

Figures for actual enrollment in mechanics alone are, however, more relevant to this discussion. Table 1 lists graduate enrollment and degrees conferred for 1966 to 1969 as reported in part two of the February issues of the ASEE journal. Data were summarized for 42 schools reporting for all four years and for six additional schools whose programs started after 1965-66. Note that the M.S. enrollment and degrees conferred have declined almost

*Presented at the 1970 ASEE Annual Meeting Ohio State University, Columbus, Ohio.

20% from their maximum. Total graduate enrollment in the three schools producing the most Ph. D's. (25% of the total) during the four-year interval fell by twice that amount. Growth of programs in other schools and lack of research stipends may have been a significant factor. Schools whose mechanics programs are combined with traditional curricular departments like civil, mechanical, and aerospace engineering are not included in Table 1.

Graduate Enrollment and Degrees Conferred in Engineering Mechanics
For 1965-69

	1965-66	1966-67	1967-68	1968-69	Maximum Change(%) 1965-69
M.S. Enrollment					
a.	753	689	643	694	
b.	753	735	759	694	-19.7
Ph.D. Enrollment					
a.	745	820	793	788	
b.	745	830	821	814	-03.9
M.S. Degrees					
a.	279	328	289	271	
b.	279	340	311	297	-17.4
Ph.D. Degrees					
a.	142	145	162	186	
b.	142	148	164	189	+23.7
M.S. + Ph.D. Enrollment					
c.	231	205	156	146	-36.8

- a. 42 schools
- b. 42 + 6 new schools
- c. 3 schools producing 25% of all Ph.D's.

Engineering mechanics departments probably have more need to develop graduate programs educating engineers for R&D assignments than the more traditional curricula. A report by the Engineers Joint Council (EJC Engineer, 1969) covered one in every seven engineers who belong to major engineering societies, and who are either registered or are college graduates or both. This survey of 53,000 engineers indicated that 30% become managers, 20% engage in research, and only 3% teach. Thus, the ratio of teachers to research engineers should be no more than one in seven, but my own experience indicates that a much higher percentage of mechanics graduates have sought and obtained academic appointments. The result has been the development of more graduate programs, which compete with those already established, although it appears at the moment to be a declining market. At the same time, all of these programs resemble each other and tend to pro-

duce stereotyped individuals seeking to perpetuate themselves on college faculties. There is a great need for practitioner-oriented engineers trained at the highest academic levels for R&D tasks in industry. Perhaps the present shortage of academic positions will force graduates to obtain the industrial experience that is so necessary for all college faculties before they hope for future appointments to educational institutions.

There is at the moment serious concern as to whether there is already an overproduction of Ph. D. 's in engineering of the type now being produced. Those in allied disciplines such as chemistry, physics, and mathematics have experienced considerable difficulty in the past two years in obtaining professional employment of their liking (U.S. News & World Report, Dec. 29, 1969).

In June 1968, 2,933 doctoral degrees in engineering were awarded. The total engineering faculty that year totaled 18,961. If we assume a 10% turnover of engineering faculty because of retirement or transfer to industry, and if 1,900 Ph. D. 's enter the teaching profession each year, industry would have to absorb the balance. Just what industry's potential needs are for research-minded engineers is still an open question. There certainly is a saturation point, but these individuals might have to reorient their professional objectives from research to design, production, and management. In this sense, engineering colleges today produce virtually no practitioners.

R&D OR P. E. ?

One possible solution to this problem of educating graduate students in mechanics, as well as in other engineering disciplines, would be to have two programs, one for research and one for the practice of engineering. This would be in line with the projections published by the University of California on future engineering educational needs. It would be necessary for the different doctor of philosophy and doctor of engineering degrees to be used for these two programs. For maximum effectiveness, it appears that the doctor of engineering degree should be administered by the college of engineering rather than by the graduate school, but that the Ph. D. research degree should remain the province of the graduate school as it is now.

The professional degree of doctor of engineering should be heavily oriented toward design, and some means should be found for providing a meaningful internship (Pletta, 1950). Thus, the destination of the bulk of graduate engineering students should be pointed toward industry rather than academia. The leading professional engineers of the future--perhaps 2% of the total number--will have doctoral training in some specialty with a broad foundation in all the engineering sciences, a solid introduction to humanities and social studies, and adequate training in management.

They will have to deal with individuals in all walks of life to design the increasingly complex systems that will be needed. Perhaps it is time that all graduate programs be oriented to the crying need for leadership in our society rather than to the continued production of specialists in ever-narrowing fields. If more schools were concerned about producing these great engineering leaders of the future and developing whole new programs for their education, we would not be concerned with perpetuating specialized curricula in what now appears to be declining enrollments.

This true professional engineer would have to take a broad interest in society and "have to study the 'laws of politics' -- and harness the over-grown giant of governing machinery which has become America's master, rather than its servant." He would wonder whether engineers would "bungle the job more than politicians have done" or whether politicians could "have done as well in bending the forces of nature to man's convenience as engineers have done?" This engineer would know that "the job of harnessing politics for the use and convenience of man is yet to be done (Mavis, 1952)."

Has not the time come when we professional engineers and our professional schools had best recognize our obligations to society or surrender our profession to governmental, industrial, or labor union bureaucracies?

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A NOTE ON THE COMPUTATION OF THE APPROXIMATE VALUE OF THE BUCKLING LOAD FOR A UNIFORM BAR WITH ELASTIC END RESTRAINTS

D. A. Ziegenbock

The case to be considered in detail is the symmetric one, with equal end restraints. The procedure described gives the approximate location of the points of counter-flexure. The buckled bar will have the shape of a half-sine curve between these points, and a curve which fits the boundary conditions is assumed for the end portion of the bar. The result of this method is a simple algebraic expression for the location of the point of counter-flexure in terms of E (modulus of elasticity), L (length), I (moment of inertia of the bar cross-section), and C (the elastic end restraint). The exact method of solution is not particularly difficult, but the solution of the differential equation gives the buckling load in a transcendental equation which must be solved by trial or by the use of a computer. The approximate value of the buckling load, using the method described here, can be corrected by the use of the included correction curve. This curve was obtained by assuming various values of the buckling load

$$P_{cr} = n \frac{\pi^2 EI}{L^2} \quad \text{where } 1 \leq n \leq 4$$

The corresponding value of C , the elastic end restraint, was then computed for each assumed value of P_{cr} , and this value was used to obtain the approximate value of the distance "b", which is the distance from the end of the bar to the point of counter-flexure. This value is then used to obtain the approximate value of the buckling load, which is finally compared to the assumed value. The correction curve also gives the maximum error of the approximate method. For many engineering computations, the approximate method is sufficiently accurate, since the physical constants are frequently not known with a high degree of accuracy.

The chief use of the method described here, however, is to assist the student of stability problems in visualizing the behavior of a buckled bar with elastic end restraints and to emphasize the similarity with a simple pin-ended column commonly discussed in a beginning course in mechanics of materials.

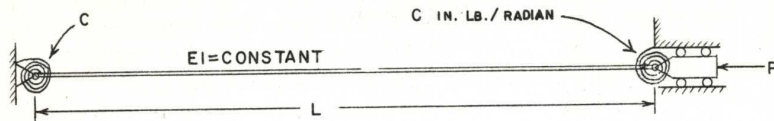


FIG. 1(a)

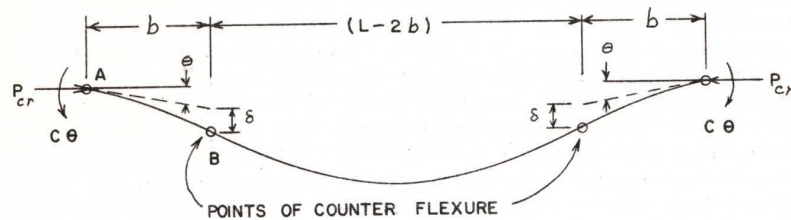
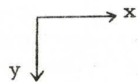


FIG. 1(b)

Sign Conventions



Positive Moment

Hence

$$EI \frac{d^2 y}{dx^2} = -M$$

For the portion AB of the bar, Fig. 1(b), the origin is at A and the assumed curve is

$$y = \theta x + \delta \left(1 - \cos \frac{\pi x}{2b}\right)$$

$$\frac{dy}{dx} = \theta + \frac{\delta \pi}{2b} \sin \frac{\pi x}{2b} \quad (= \theta \text{ when } x = 0)$$

$$\frac{d^2 y}{dx^2} = \frac{\delta \pi^2}{4b^2} \cos \frac{\pi x}{2b} \quad (= 0 \text{ when } x = b)$$

$$\text{At } x = 0: \frac{d^2 y}{dx^2} = \frac{\delta \pi^2}{4b^2}$$

Therefore

$$EI \frac{\delta \pi^2}{4b^2} = -(-C\theta) = C\theta$$

and

$$\delta = \frac{4b^2 C}{\pi^2 EI} \theta$$

Also, considering AB as a free body, and since the moment at point B is zero

$$P'_{cr} (b\theta + \delta) - C\theta = 0$$

where P'_{cr} is the approximate value of the buckling load, as distinguished from P_{cr} , which is the correct value of the buckling load.

Therefore:

$$C\theta = P'_{cr} \left(b\theta + \frac{4b^2 C}{\pi^2 EI} \theta\right)$$

and

$$C = P'_{cr} \left(b + \frac{4b^2 C}{\pi^2 EI}\right)$$

However, using Euler's equation for pin-ended columns

$$P'_{cr} = \frac{\pi^2 EI}{(L-2b)^2}$$

$$C = \frac{\pi^2 EI}{(L-2b)^2} \left(b + \frac{4b^2 C}{\pi^2 EI}\right) = \frac{1}{(L-2b)^2} (\pi^2 EIb + 4b^2 C)$$

$$C(L^2 - 4Lb + 4b^2) = \pi^2 EIb + 4b^2 C$$

Solving for b

$$b = \frac{CL^2}{\pi^2 EI + 4LC} = \frac{L^2}{\left(\frac{\pi^2 EI}{C} + 4L\right)}$$

For pinned ends: $C = 0$ and $b = 0$ (correct value)
 For fixed ends: $C = \infty$ and $b = \frac{L}{4}$ (correct value)
 For intermediate values of C , the values of "b" obtained from the above equation will be lower than the correct value, and hence $(L - 2b)$ is larger than the correct distance between the points of counter-flexure, and therefore $P'_{cr} < P_{cr}$. Fig. 2 shows the relationship between the percentage error and the approximate value of the buckling load, P'_{cr} . The error is always less than 3%, which is adequate for many engineering applications. For those instances where the physical constants are known with sufficient accuracy, Fig. 2 can be used to obtain P_{cr} .
 $P_{cr} = P'_{cr} (1 + e)$, where e = error.

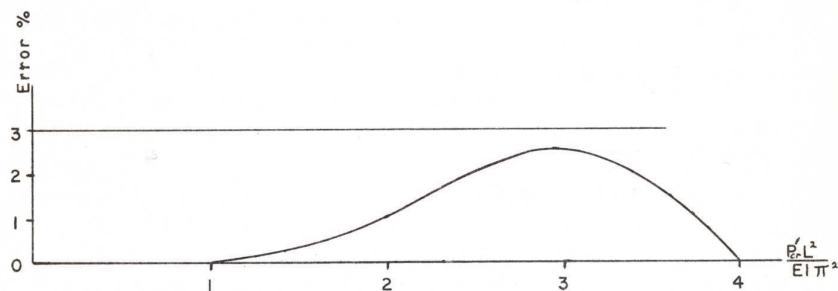


Fig. 2 was obtained in the following way. The solution of the differential equation is

$$\tan \frac{KL}{2} = - \frac{P_{cr}}{CK},$$

where

$$K^2 = \frac{P_{cr}}{EI}; \quad P_{cr} = n \frac{\pi^2 EI}{L^2}$$

where

$$1 \leq n \leq 4$$

Therefore

$$K^2 = n \frac{\pi^2}{L^2} \quad \text{and} \quad K = \frac{\pi}{L} \sqrt{n}; \quad \frac{KL}{2} = \frac{\pi}{2} \sqrt{n}$$

$$\frac{P_{cr}}{K} = \frac{n \frac{\pi^2 EI}{L^2}}{\frac{\pi}{L} \sqrt{n}} = \left(\frac{\pi EI}{L} \right) \sqrt{n}$$

Solving the first equation for C

$$C = - \frac{\left(\frac{\pi EI}{L} \right) \sqrt{n}}{\tan \frac{\pi}{2} \sqrt{n}}$$

Assuming a value of n , the corresponding value of C is computed. This value of C is then used to obtain the approximate value of b and then the value of P'_{cr} which is compared with P_{cr} . The process is repeated to obtain a number of points on the graph of Fig. 2.

A similar method can be used for the case with an elastic restraint at one end, while the other end is pinned. There is much less computational advantage for this case, however. If "a" represents the distance from the elastically restrained end to the point of counter-flexure, the following equation containing "a" results

$$(L - a)^3 = \frac{La}{C} \pi^2 EI + 4La^2 \quad \text{and} \quad P'_{cr} = \frac{\pi^2 EI}{(L-a)^2}$$

Since the solution of a cubic equation may be about as tedious as the solution of a transcendental equation, the unsymmetrical case is not discussed in detail.